Common Operations on Signals

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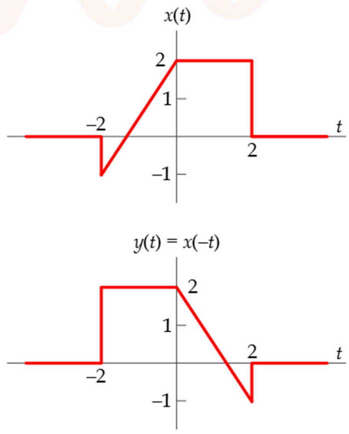


Common operations on signals:

Operation	Equation
Amplitude scaling	$y(t) = a\,x(t)$
Amplitude shifting	$y(t) = x(t) + b$
Amplitude reversal (inversion)	$y(t) = -x(t)$
Time scaling	$y(t) = x(t/\tau)$
Time shifting	$y(t) = x(t + t_0)$
Time reversal (inversion)	$y(t) = x(-t)$

Also called signal transformations. To avoid any possible confusion, we keep the word transform for Fourier transform, Laplace transform and z-transform. These operations will be very useful in later discussion.

Amplitude scaling

$$y(t) = a\,x(t)$$

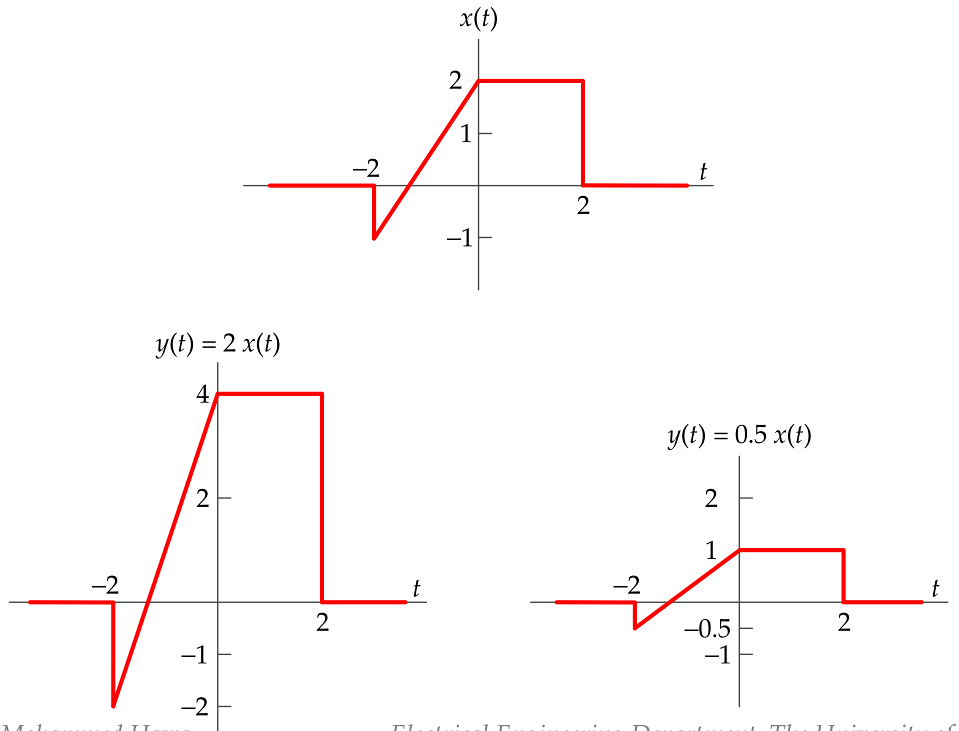
where a is a positive constant.

Each value in $x(t)$ is **multiplied** by the same constant a.

When $a > 1$, this **stretches (expands)** the signal vertically (i.e., across the vertical axis), but the shape remains similar to the original shape.

When $a < 1$, this **shrinks (compresses)** the signal vertically (i.e., across the vertical axis), but the shape remains similar to the original shape.

This occurs, for example, during amplification of signals by electronic amplifiers and attenuation of signals inside communication channels.



Amplitude shifting

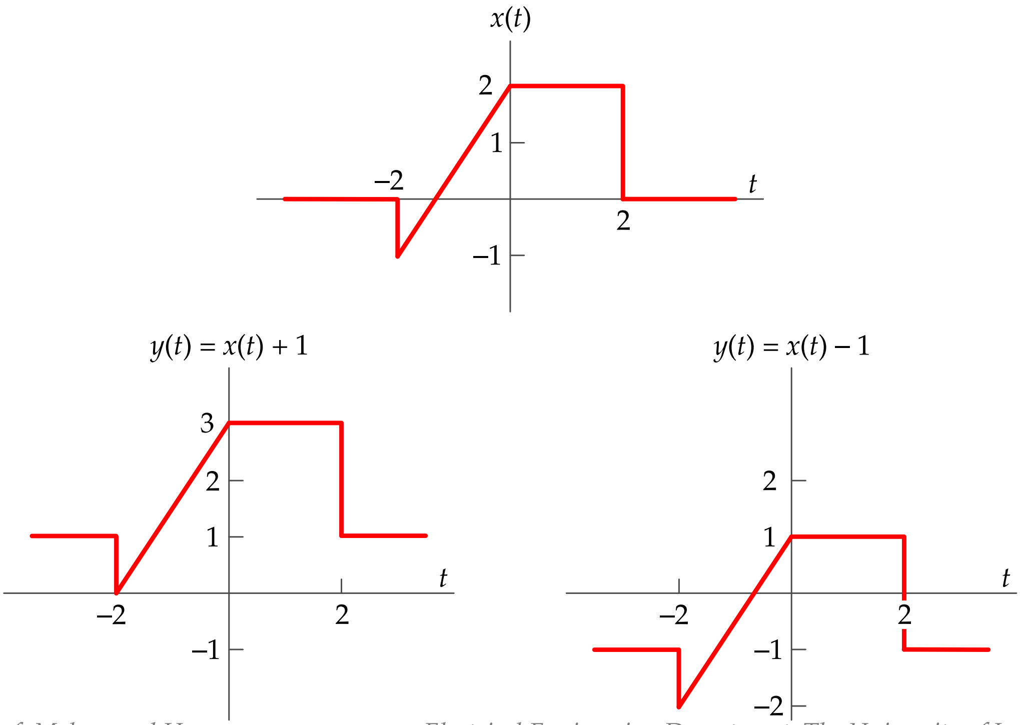
$$y(t) = x(t) + b$$

where b is a constant, called offset or bias or DC shift.

An extra constant b gets **added** to each value in $x(t)$. All values of $x(t)$ get the same exact extra value of b.

When $b > 0$ [positive], this shifts the signal **up** (vertically), but the shape remains the same as the original shape.

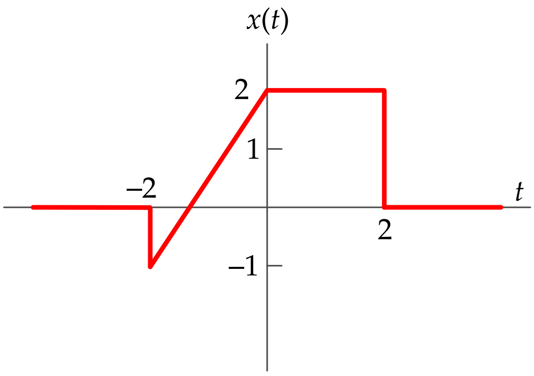
When $b < 0$ [negative], this shifts the signal **down** (vertically), but the shape remains the same as the original shape.

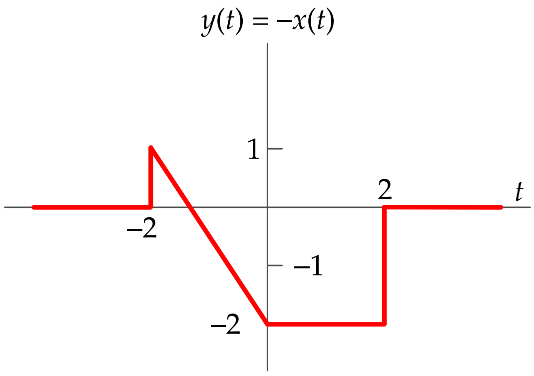


Amplitude reversal (inversion) [special case of amplitude scaling]

$$y(t) = -x(t) = -1\,x(t)$$

All positive values of $x(t)$ become negative and all negative values of $x(t)$ become positive. This is a reflection of $x(t)$ (via a mirror) about the horizontal axis (a flip or mirror-image about the horizontal axis).





Multiple operations combined

$$y(t) = -a\,x(t) + b$$

Pay attention to order: Perform amplitude reversal first, followed by amplitude scaling and finally perform the amplitude shifting (similar to how operator precedence works in programming).

Amplitude reversal (multiplication by -1) and amplitude scaling is commutative (order can be switched). But avoid switching the order of amplitude scaling and amplitude shifting, unless you adjust the shift appropriately.

Time scaling

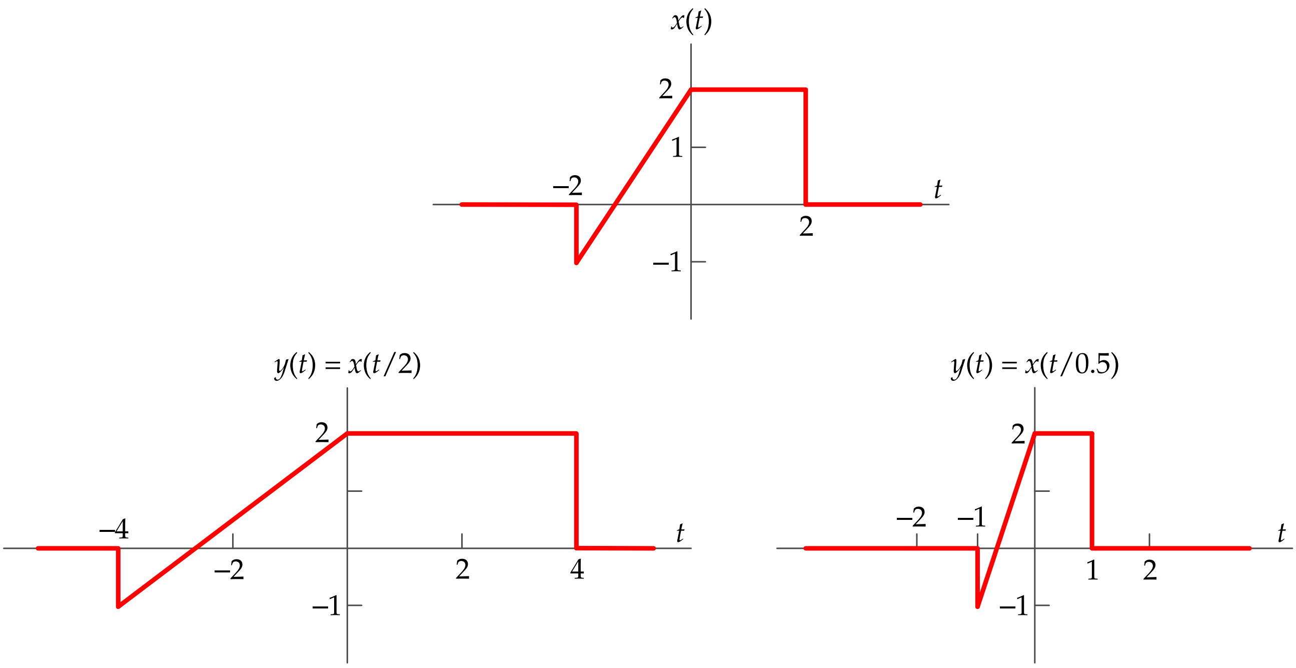
$$y(t) = x\left(\frac{t}{\tau}\right)$$

where τ is a positive real constant.

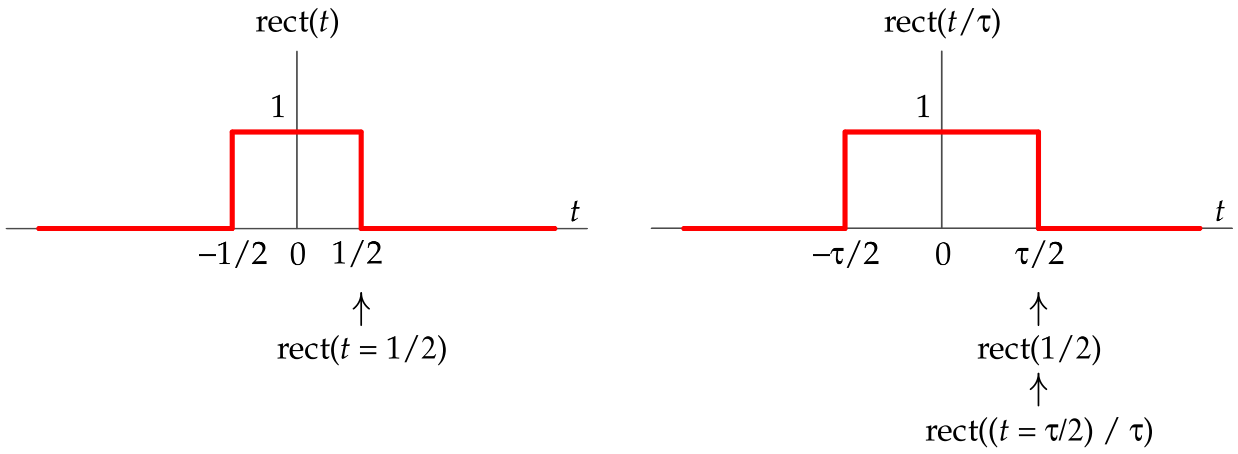
The time variable itself shrinks by a factor τ. Hence, without affecting the amplitude:

When $\tau > 1$, this **stretches (expands)** the signal horizontally (i.e., across the horizontal axis), but the shape remains similar to the original shape.

When $\tau < 1$, this **shrinks (compresses)** the signal horizontally (i.e., across the horizontal axis), but the shape remains similar to the original shape.



Reason: You need to substitute scaled up time t instances to get the same original $x(t)$ values



Easy to remember: τ is the pulse width for $\text{rect}(t/\tau)$

For sinusoidal signals, this is similar to the all-familiar concept of fundamental period and fundamental frequency

$$x(t) = A\cos(\omega_0 t) = A\cos\left(\frac{t}{\tau}\right)$$

$$\omega_0 = \frac{1}{\tau} = \frac{2\pi}{T_0}$$

$\tau \uparrow$, $\omega_0 \downarrow$, $T_0 \uparrow$, expansion

$\tau \downarrow$, $\omega_0 \uparrow$, $T_0 \downarrow$, compression

In time scaling the vertical axis is the anchor point, which remains unchanged under the scaling operation because at $t = 0$, we have

$$x(t/\tau) = x(0/\tau) = x(0) = x(t)$$

Notice that if the signal $y(t)$ is an expansion of $x(t)$ because

$$y(t) = x\left(\frac{t}{\tau}\right)$$

Then the signal $x(t)$ is a compression of $y(t)$ because

$$y(\tau\, t) = x(t)$$

Hint: Change variables using $\zeta = t/\tau$, so that $t = \tau\,\zeta$

Time shifting

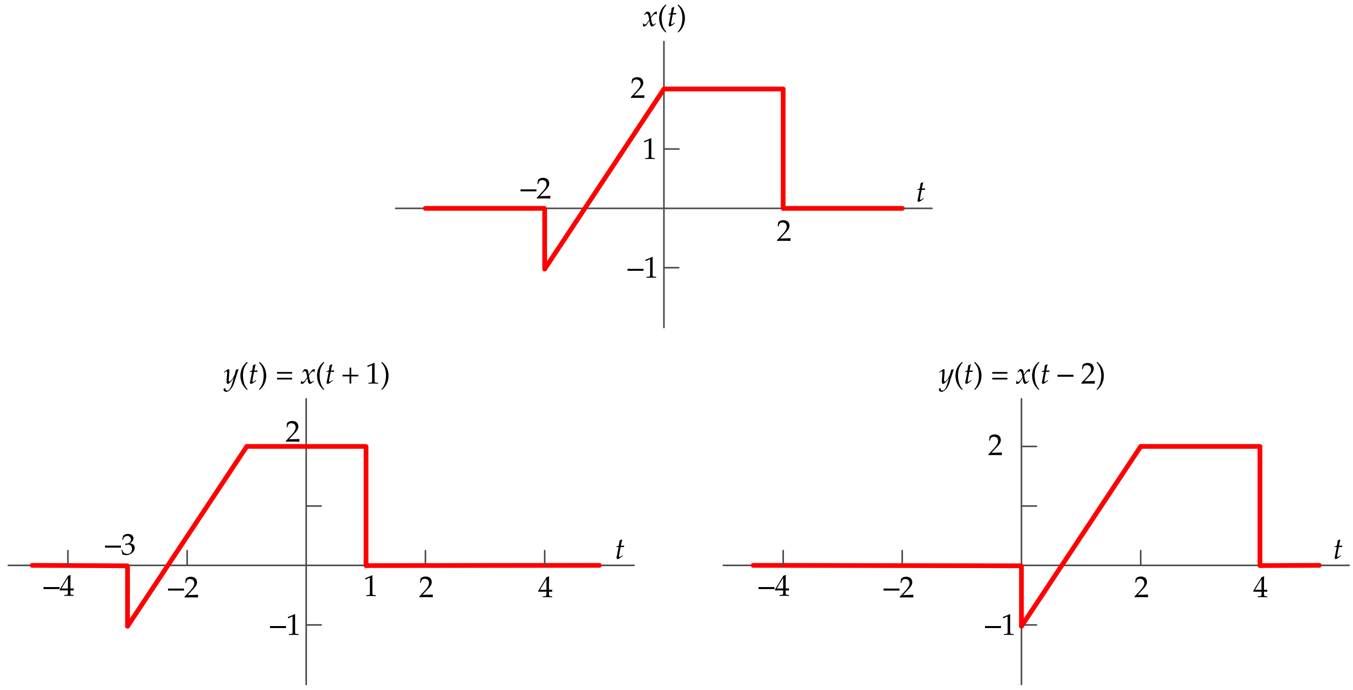
$$y(t) = x(t + t_0)$$

where t_0 is a real constant.

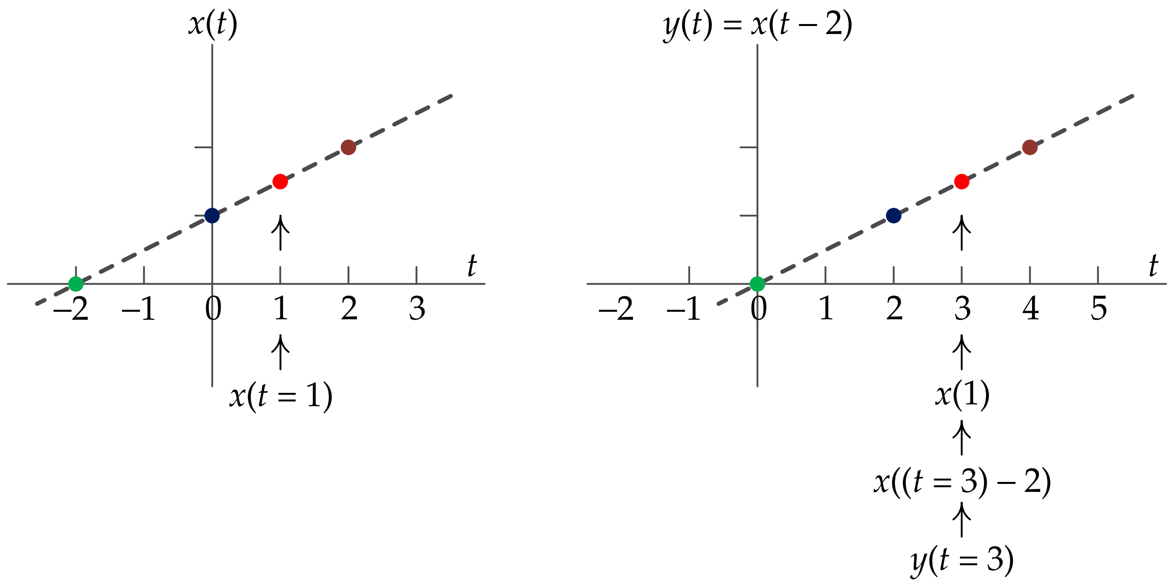
When $t_0 > 0$ [positive], this shifts the signal **to the left** (horizontally) [leading or advanced signal], but the shape remains the same as the original shape.

When $t_0 < 0$ [negative], this shifts the signal **to the right** (horizontally) [delayed or lagging signal], but the shape remains the same as the original shape.

Notice you need smaller times t to get the original $x(t)$ values, so you get the time advance.



Reason: For negative t_0, you need to substitute bigger t instants to get the same original $x(t)$ values



Notice that if the signal $y(t)$ leads $x(t)$ because

$$y(t) = x(t + t_0)$$

Then the signal $x(t)$ lags $y(t)$ because

$$y(t - t_0) = x(t)$$

Hint: Change variables using $\zeta = t + t_0$, so that $t = \zeta - t_0$

Notice the similarity with the concept of phase shift for sinusoidals:

$$x(t) = A\cos(\omega_0 t + \varphi) = A\cos\big(\omega_0(t + t_0)\big) = A\cos(\omega_0 t + \omega_0 t_0)$$

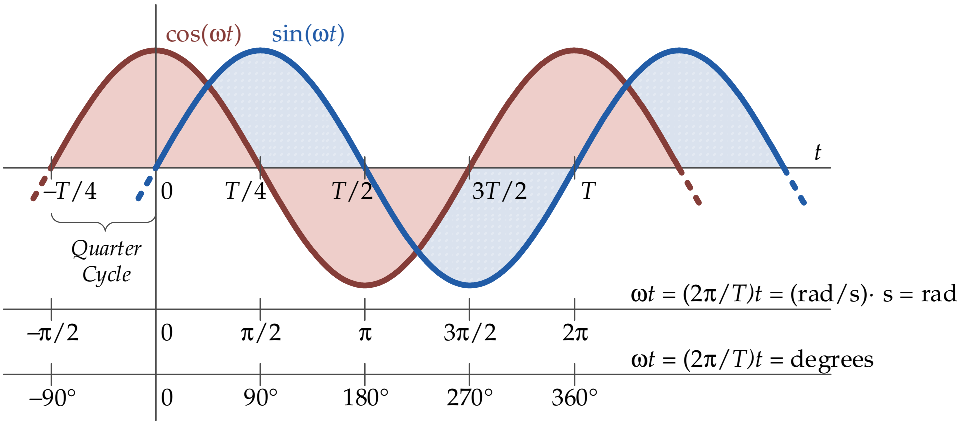
$$\varphi = \omega_0 t_0 = 2\pi \times \frac{t_0}{T_0}$$

$t_0 = 0$ seconds time shift → $\varphi = 2\pi \times \frac{0}{T_0} = 0$ radians phase shift.

$t_0 = T_0$ seconds time shift → $\varphi = 2\pi \times \frac{T_0}{T_0} = 2\pi$ radians phase shift (one cycle shift).

$t_0 = \frac{T_0}{2}$ seconds time shift → $\varphi = 2\pi \times \frac{T_0/2}{T_0} = \frac{2\pi}{2} = \pi$ radians phase shift (half cycle shift), etc.

Think about **horizontal** shift as either **time shift** (sec), or **phase shift** φ (in radians or in degrees). One cycle is T sec or 2π rad.



$$A\sin(\omega_0 t) = A\cos(\omega_0 t - \pi/2\,) = A\cos(\omega_0(t - T_0/4)\,)$$

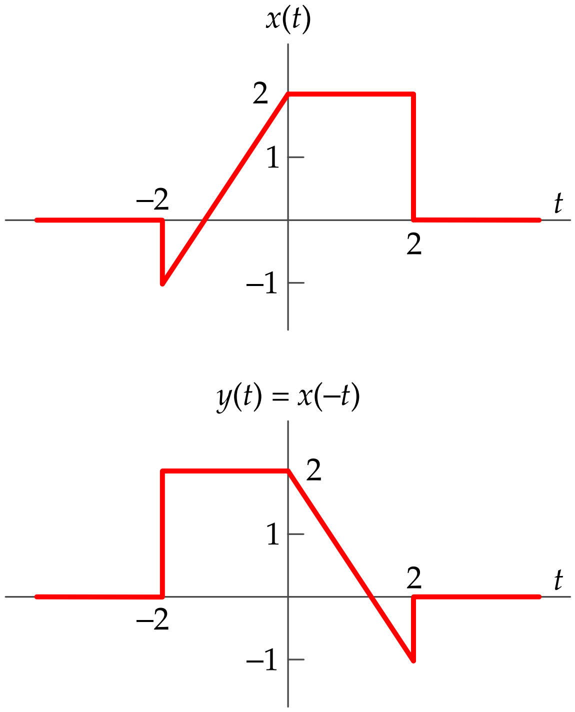
$$A\cos(\omega_0 t) = A\sin(\omega_0 t + \pi/2\,) = A\sin(\omega_0(t + T_0/4)\,)$$

Time reversal (inversion) [special case of time scaling]

$$y(t) = x(-t)$$

All values of $x(t)$ for positive time become values of $x(t)$ for negative time and all values of $x(t)$ for negative time become values of $x(t)$ for positive time. This is a reflection of $x(t)$ (via a mirror) about the vertical axis (a flip or mirror-image about the vertical axis).

Even function are unaffected by time reversal because $x(-t) = x(t)$ already (symmetry around vertical axis), while for odd functions the time reversal is the same as amplitude reversal because $x(-t) = -x(t)$.



Multiple operations combined

$$y(t) = x\left(\frac{-(t + t_0)}{\tau}\right)$$

Pay attention to order: Perform time reversal first, followed by time scaling and finally perform time shifting (time shifting t_0 is inside the brackets and is directly related to the time variable).

To avoid confusion, **always** rewrite the equation to get the above form, then proceed. Notice how the time shift becomes crystal clear in the following example

$$y(t) = x(10 - 5t) = x\left(\frac{-\{t + (-2)\}}{0.2}\right)$$

The operations discussed above for time (scaling, shifting, and reversal) also apply to functions of other independent variables (e.g., frequency).

Exercises

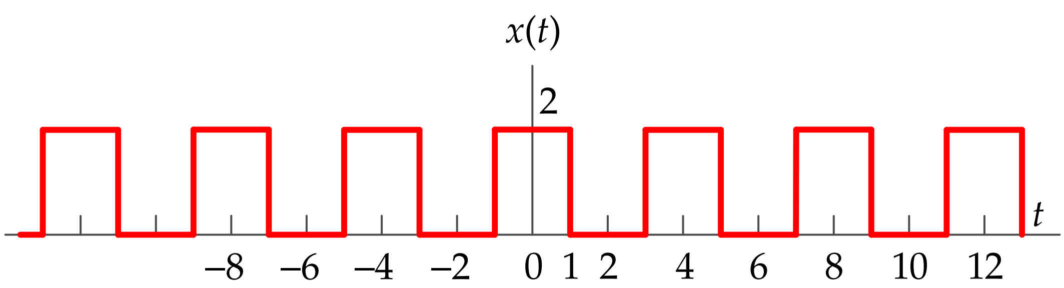
Q1. Sketch the signal $x(t) = 3\,\Delta(-0.5t + 1)$. Find the area under $x(t)$?

Q2. Sketch the signal $x(t) = -0.2\,\text{saw}\big(2(t-3)\big)$. Determine the total energy E_x in $x(t)$?

Q3. Sketch the signal $x(t) = 5 + 10\cos\left(0.5\pi(t+2) - \frac{\pi}{3}\right)$. Determine the average power P_x in $x(t)$? Can you use *superposition* for power?

Q4. Sketch the signal $x(t) = -10\,\delta(1-t)$. What is the minimum and maximum amplitude of the signal $x(t)$?

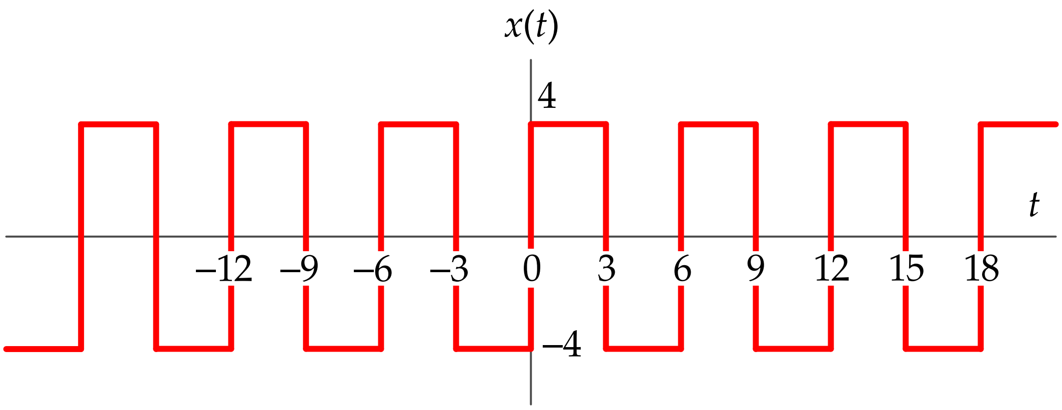
Q5. Write the following signal $x(t)$ in terms of the basic signal $\text{rect}(t)$.



Answer:

$$x(t) = \text{rep}_4\left\{2\,\text{rect}\left(\frac{t}{2}\right)\right\}$$

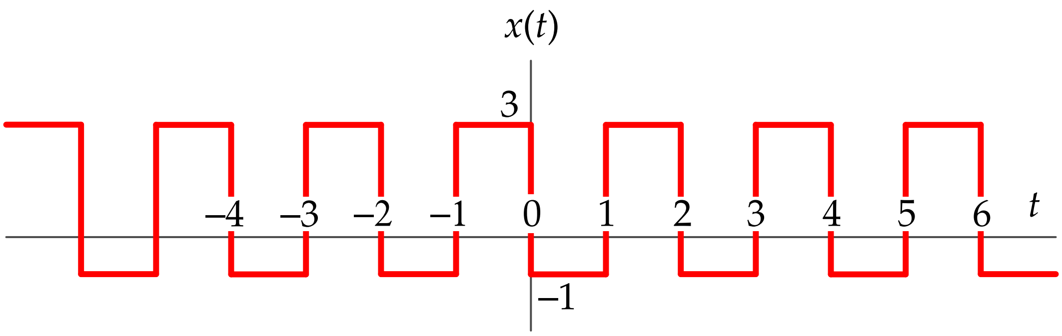
Q6. Write the following signal $x(t)$ in terms of the basic signal $\text{rect}(t)$.



Answer:

$$x(t) = \text{rep}_6\left\{8\,\text{rect}\left(\frac{t-1.5}{3}\right)\right\} - 4 = \text{rep}_6\left\{8\,\text{rect}\left(\frac{t-1.5}{3}\right) - 4\right\}$$

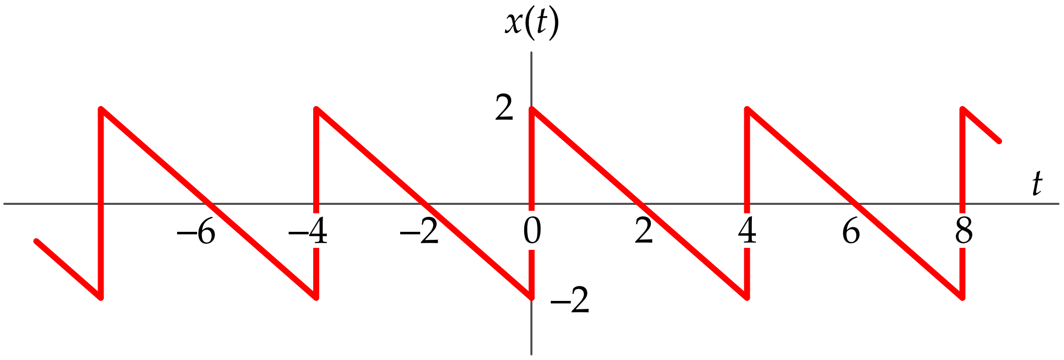
Q7. Write the following signal $x(t)$ in terms of the basic signal $\text{rect}(t)$.



Answer:

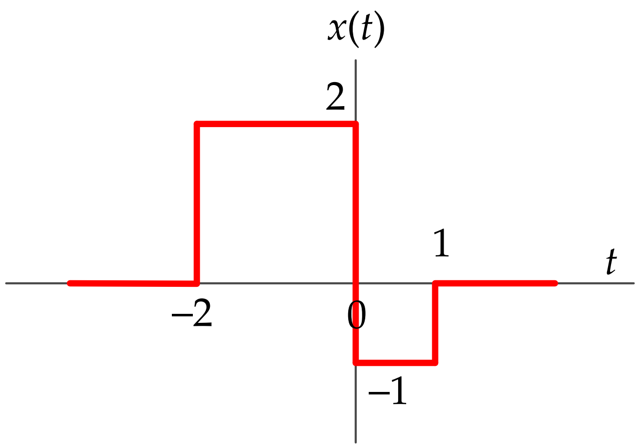
$$x(t) = \text{rep}_2\{4\,\text{rect}(t+0.5)\} - 1 = \text{rep}_2\{-4\,\text{rect}(t-0.5)\} + 3$$

Q8. Write the following signal $x(t)$ in terms of the basic signal saw(t).



Hint: Notice the slope of the linear line.

Q9. Write the following signal $x(t)$ in terms of the basic signal $u(t)$.



Hint: You need three (time-shifted) unit step signals.

- Always verify your answers above by substituting a certain time instant (say $t_1 = 2$ seconds) and reading the values from the original definition of the signal.
- Review the basic signals we investigated earlier: t, t^2, αe^{kt}, $A\cos(\omega t)$, $A\sin(\omega t)$, $u(t)$, $\text{ramp}(t)$, $\text{rect}(t)$, $\Delta(t)$, $\alpha e^{-(t^2/2)}$, $\delta(t)$, $\text{sinc}(t)$, etc, and see how amplitude/time scaling, shifting and inversion affect such signals.
- Solve more problems. The exercises here are not enough.
- The more problems you solve, the easier it gets.
- Solve as many problems as you can, not only for this lecture but for ALL other lectures.